Pre-Calculus CP 1 – Section 8.3/8.4 Notes **Name: ___________________**

Determinants, Inverses, and the Identity Matrix

Determinants are used to help you find the INVERSE of a matrix, and the inverse of a matrix will help you solve a system of equations!

The notation for a determinant looks like the absolute value notation:

$$\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} \text{ means find the determinant for matrix } \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

Here's the formula for a 2x2:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Let's try a few determinants before we see how the determinants are used.

Ex.1) $\begin{vmatrix} 7 & 2 \\ 2 & 3 \end{vmatrix} =$

Ex. 2) $\begin{vmatrix} -5 & 1 \\ -7 & 4 \end{vmatrix} =$

A 3x3 is a bit more complicated. Let's do these by hand so you will appreciate the calculator magic that much more:

Ex. 3) $\begin{vmatrix} 4 & 3 & 1 \\ 5 & -7 & 0 \\ 1 & -2 & 2 \end{vmatrix} =$

Ex.4) $\begin{vmatrix} 2 & -1 & 3 \\ -2 & 0 & 1 \\ 1 & 2 & 4 \end{vmatrix} =$

Pre-Calculus CP 1 – Section 8.3/8.4 Notes
Determinants, Inverses, and the Identity Matrix

Determinants on the Calculator!!

Keystrokes:

1. 2^{nd} matrix
2. arrow to the right for MATH
3. #1 det(
4. 2^{nd} matrix
5. enter the matrix letter that you want to find the determinant of (probably A)

Use your calculator to find the following determinants:

Ex.1) $\begin{vmatrix} -5 & 1 \\ -7 & 4 \end{vmatrix} =$

Ex. 2) $\begin{vmatrix} 4 & 3 & 1 \\ 5 & -7 & 0 \\ 1 & -2 & 2 \end{vmatrix} =$

Using the determinant:

Use #1: Find the area of a triangle: the formula is $\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

where the coordinates of the triangle go in the positions with x and y.

Example: Find the area of a triangle using the determinant formula and the coordinates (1,2), (6,2) and (4,0):

$$\begin{vmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{vmatrix}$$

You try: Find the area of a triangle using the determinant formula and the coordinates (3,9), (4,-2) and (0,5):

Pre-Calculus CP 1 – Section 8.3/8.4 Notes
Determinants, Inverses, and the Identity Matrix

Use #2: Finding an INVERSE

The inverse of a 2 x 2 matrix, $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, can be found using the following fomula:

$$A^{-1}=\frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Examples- find the inverse of the following by hand:

1) $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

2) $\begin{bmatrix} -4 & 3 \\ -3 & 2 \end{bmatrix}$

To find the inverse on the graphing calculator, input your matrix into matrix A, then press:

1. 2^{nd} matrix
2. enter for A
3. x^{-1} button
4. enter

We will not do 3 x 3 inverse matrices by hand- instead we will do them on the calculator!
Try these in your calculator (hit math-frac if you get crazy decimals) :

3) $\begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & -1 \\ -1 & 3 & -3 \end{bmatrix}$

4) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & -1 & -2 \\ 3 & 1 & 1 \end{bmatrix}$

Pre-Calculus CP 1 – Section 8.3/8.4 Notes
Determinants, Inverses, and the Identity Matrix

The IDENTITY matrix:

The identity matrix is like multiplying by 1 – if you multiply a matrix by the identity, it will stay unchanged. Identity matrices can only be square, or n x n. Two examples are:

2 x 2: 3 x 3:

If you ever want to check to ensure that you have the right inverse matrix,

Two matrices are INVERSES of each other if their **product** is the identity matrix

Ex: $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} \_\_\_\_\_\_ & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ \end{bmatrix} = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$

Ex: Are the following two matrices inverses of each other? Find the product to prove your answer:

$$\begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1.5 & -3 \end{bmatrix}$$

HW: p. 608 #1, 3, 5, 13, 17, 21, 31 **and** p. 616 # 5, 15, 21 **and** p. 628 #15, 29